2. Numbers and Sequences

1 mark Questions

1. Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy
   (A) $1 < r < b$ (B) $0 < r < b$ **(C) $0 \leq r < b$** (D) $0 < r \leq b$
2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are PTA-5, SEP-20
   (A) 0, 1, 8 (B) 1, 4, 8 (C) 0, 1, 3 (D) 1, 3, 5
3. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
   (A) 4 **(B) 2** (C) 1 (D) 3 MAY-22
4. The sum of the exponents of the prime factors in the prime factorization of 1729 is
   (A)1 (B) 2 **(C) 3** (D) 4 SEP-21,PTA-4,JUL-22
5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
   (A) 2025 (B) 5220 (C) 5025 **(D) 2520**
6. $7^{4k} \equiv$ _____$(mod\ 100)$ PTA-1
   (A) 1 (B) 2 (C) 3 (D) 4
7. Given $F_1 = 1, F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is SEP-21, MDL
   (A) 3 (B) 5 (C) 8 **(D) 11**
8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.
   (A) 4551 (B) 10091 **(C) 7881** (D) 13531
9. If 6 times of 6^{th} term of an A.P is equal to 7 times the 7^{th} term, then the 13^{th} terms of the A.P is
   (A) 0 (B) 6 (C) 7 (D) 13 PTA-4
10. An A.P consists of 31 terms. It is 16^{th} term is m, then the sum of all the terms of this A.P is
    (A) 16m (B) 62m **(C) 31m** (D) $\frac{31}{2}$ m PTA-5
11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120? MDL
    (A) 6 (B) 7 **(C) 8** (D) 9
12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \cdots + 2^0$ which of the following is true? PTA-6, SEP-20
    (A) B is 2^{64} more than A (B) A and B are equal
    (C) B is larger than A by 1 **(D) A is larger than B by 1**
13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \ldots$ is PTA-2
    (A) $\frac{1}{24}$ **(B) $\frac{1}{27}$** (C) $\frac{2}{3}$ (D) $\frac{1}{81}$
14. If the sequence $t_1, t_2, t_3, \ldots$ are in A.P then the sequence $t_6, t_{12}, t_{18}, \ldots$ is
    (A) a Geometric Progression
    (B) an Arithmetic Progression
    (C) neither an Arithmetic Progression nor a Geometric Progression
    (D) a constant sequence
15. The value of $(1^3 + 2^3 + 3^3 + \cdots + 15^3) - (1 + 2 + 3 + \cdots + 15)$ is PTA-3
    (A) 14400 (B) 14200 **(C) 14280** (D) 14520

2 mark Questions

1. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

PTA-1

Given: A man has 532 flower pots.

Each row contains 21 flower pots.

Thus, Dividend = 532

Divisor = 21

By Euclid division lemma,

$532 = 21(25) + 7$

```
      25
21 | 532
     42
     112
     105
       7
```

$a = bq + r\ ,\ 0 \le r < b$

The quotient = 25, remainder= 7

∴ **25 rows** are completed and

7 flower pots are left over

2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?

SEP-20

Given: $m, n \in N$ and $2^n \times 5^m$

$$n = 1, m = 1 \Rightarrow 2^1 \times 5^1 = 2 \times 5 = 10$$

$n = 1, m = 2 \Rightarrow 2^1 \times 5^2 = 2 \times 25 = 50$

$n = 2, m = 3 \Rightarrow 2^2 \times 5^3 = 4 \times 125 = 500$

∴ 2^n is always even.

So that, the product of 5 is in always end digit is 0.Hence, **No value** of $2^n \times 5^m$ end with the digit 5.

3. If $13824 = 2^a \times 3^b$ then a and b.

MAY-22

Given $13824 = 2^a \times 3^b$

The number 13824 can be factorized

As, $13824 = 2^9 \times 3^3$

Hence, $2^a \times 3^b = 2^9 \times 3^3$

∴ $a = 9$ and $b = 3$

```
3 | 13824
3 | 4608
3 | 1536
2 | 512
2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
    2
```

4. Find the least number that is divisible by the first ten natural numbers.

JUL-22

The first ten natural numbers are, 1,2,3,4,5,6,7,8,9,10.

Given: the number is divisible by first ten natural numbers.

Thus, LCM of 1,2,3,4,5,6,7,8,9, and 10

$= 1 \times 2^3 \times 3^2 \times 5 \times 7$

$= 8 \times 9 \times 35$

$= 2520$

∴ The least number is **2520**

5. **If x is congruent to 13 modulo 17 then $7x - 3$ is congruent to which number modulo 17 ?** PTA-2

Given: $x \equiv 13 \ (mod 17)$ [If $a \equiv b(mod\ m)$ then $a \times c \equiv b \times c\ (mod\ m)$]

Multiply by 7

$7x = 91\ (mod\ 17)$

$$17 \overline{)88} = 5,\quad 88 - 85 = 3$$

$7x - 3 \equiv 91 - 3\ (mod\ 17)$

$7x - 3 \equiv 88(mod\ 17)$

$7x - 3 \equiv 3\ (mod\ 17)$ [$\because 88 \equiv 3\ (mod\ 17)$]

$\therefore$ $7x - 3$ is congruent to **3 modulo 17.**

6. **Find the 19^{th} term of an $A.P.$ $-11, -15, -19, \ldots$** MDL, JUL-22

Given, $A.P$ is $-11, -15, -19, \ldots$

$a = -11, d = t_2 - t_1 = -15 + 11$

$d = -4$

n^{th} term of $A.P$ $t_n = a + (n-1)d$

$n = 19 \Rightarrow t_{19} = -11 + (19-1)(-4)$

$= -11 + 18(-4)$

$= -11 - 72$

$t_{19} = -83$

7. **Which term of an A. P. $16, 11, 6, 1, \ldots$ is -54 ?** MAY-22

Given: $A.P$ is 16,11,6,1, …

$t_n = -54$, $a = 16,$

$d = t_2 - t_1 = 11 - 16 = -5$

$n = \left(\frac{l-a}{d}\right) + 1$

$= \left(\frac{-54-16}{-5}\right) + 1 \quad = \left(\frac{-70}{-5}\right) + 1$

$n = 14 + 1 = 15$

$\therefore$ $\boldsymbol{t_{15} = -54}$

8. **Find the middle term(s) of an $A.P.$ $9, 15, 21, 27, \dots, 183$.** PTA-1

Given: $A.P$ is $9,15,21,27,\dots 183$.

$a = 9, \quad d = t_2 - t_1 = 15 - 9$

$l = 183\,, \; d = 6$

The number of term in $A.P$.

$n = \frac{l-a}{d} + 1$

$= \frac{183-9}{6} + 1 = \frac{174}{6} + 1$

$= 29 + 1$

$n = 30$

$n = 30$ even,

The middle term $= \frac{n^{th}}{2}$ term and $\left(\frac{n}{2} + 1\right)^{th}$ term

$= \frac{30}{2}$ term and $\frac{30}{2} + 1$ term

$= 15^{th}$ term and 16^{th} term

$t_n = a + (n-1)d$

$n = 15 \Rightarrow$

$t_{15} = 9 + (15-1)(6) = 9 + (14)(6)$

$t_{15} = 93$

$n = 16 \Rightarrow$

$t_{16} = 9 + (16-1)(6) = 9 + (15)(6)$

$t_{16} = 99$

$\therefore$ The middle terms are $\boldsymbol{t_{15} = 93}$, $\boldsymbol{t_{16} = 99}$

9. **If $3 + k, 18 - k, 5k + 1$ are in $A.P.$ then find k.** SEP-21, PTA-3, 5

Given: $3 + k, 18 - k, 5k + 1$ are in $A.P$.

ie, $d = t_2 - t_1 = t_3 - t_2$

$18 - k - (3 + k) = 5k + 1 - (18 - k)$

$18 - k - 3 - k = 5k + 1 - 18 + k$

$15 - 2k = 6k - 17$

$15 + 17 = 6k + 2k$

$32 = 8k$

$k = 4$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each, successive row contains two additional seats than its front row. How many seats are there in the last row?

PTA-4

Given: 30 rows were allotted in the theatre

$n = 30$

20 seats in the front row then $a = 20$

2 seats increased in each row.

Thus, $1^{st}, 2^{nd}, 3^{rd}, \ldots 30$ rows are $20, 22, 24, \ldots$ respectively.

It is an $A.P.$ $d = t_2 - t_1 = 22 - 20 = 2$

To find: t_{30}

$t_n = a + (n-1)d$

$t_{30} = 20 + (30-1)2$

$= 20 + (29)(2)$

$= 20 + 58$

$t_{30} = 78$

78 seats in the last row.

5 mark Questions

1. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

JUL-22

Given, 1230 and 1926 leaving remainder 12 in each case, when divided by largest number.

$1230 - 12 = 1218$ and $1926 - 12 = 1914.$

Let $a = 1914$ and $b = 1218$ $a > b$

By using Euclid's lemma, $a = bq + r, 0 \le r < b$

$1914 = 1218(1) + 696$

The remainder $696 \neq 0$

$1218 = 696(1) + 522$

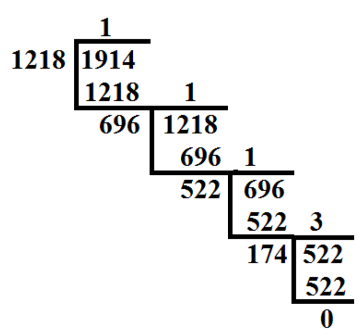
The remainder $522 \neq 0$

$696 = 522\,(1) + 174$

The remainder $174 \neq 0$

$522 = 174\,(3) + 0$

The remainder is 0



$\therefore$ The largest number is **174** which divides 1230 and 1926 and leaves remainder 12.

2. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

JUL-22

Given:

$9t_9 = 15t_{15} \quad (\because t_n = a + (n-1)d)$

$9[a + (9-1)]d = 15[a + (15-1)d]$

$9(a + 8d) = 15\,(a + 14d)$

$9a + 72d = 15a + 210d$

$9a - 15a + 72d - 210d = 0$

$-6a - 138d = 0$

$-6(a + 23d) = 0$

$6[a + (24-1)d] = 0$

$6t_{24} = 0$

Hence proved.

3. The sum of three consecutive terms that are in $A.P.$ is 27 and their product is 288. Find the three terms.

SEP-21

Let the three consecutive terms be

$a-d,\ a,\ a+d$

Given: $a - d + a + a + d = 27$

$3a = 27$

$a = 9$

Also, $(a-d)(a)(a+d) = 288$

$(a^2 - d^2)a = 288$

$(9^2 - d^2) = \frac{288}{9}$

$81 - d^2 = 32$

$-d^2 = 32 - 81$

$-d^2 = -49$

$d = \pm 7$

When $a = 9, d = 7$ the $A.P$ is

$9-7,\ 9,\ 9+7$

2, 9, 16

When $a = 9, d = -7$

$9+7,\ 9,\ 9-7$

16, 9, 2.

4. **The ratio of 6^{th} and 8^{th} term of an $A.P.$ is $7:9$. Find the ratio of 9^{th} term to 13^{th} term.**

MAY-22

Given: $t_6 : t_8 = 7:9 \Rightarrow \frac{t_6}{t_8} = \frac{7}{9}$

$\frac{a+(6-1)d}{a+(8-1)d} = \frac{7}{9}$ $[\because t_n = a + (n-1)d]$

$9(a + 5d) = 7(a + 7d)$

$9a + 45d = 7a + 49d$

$9a - 7a = 49d - 45d$

$2a = 4d$

$a = 2d$ ....(1)

To find, $t_9 : t_{13} = \frac{t_9}{t_{13}}$

$= \frac{a+(9-1)d}{a+(13-1)d}$

$= \frac{a+8d}{a+12d}$

$= \frac{2d+8d}{2d+12d}$

$= \frac{10d}{14d} = \frac{5}{7}$

$\therefore \boldsymbol{t_9 : t_{13} = 5:7}$

5. **A man joined a company as Assistant Manager. The company gave him a starting salary of ₹60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?**

PTA-6

The starting salary of man is ₹ 60,000

His salary increased 5% annually.

$\mathrm{P} = 60000$, $r = 5\%$, $n = 5$ years

$A = P\left(1 + \frac{r}{100}\right)^n$

$= 60000\left(1 + \frac{5}{100}\right)^5$

$= 60000\left(\frac{21}{20}\right)^5$

$= 60000\left(\frac{21\times21\times21\times21\times21}{20\times20\times20\times20\times20}\right)$

$= \frac{12252303}{160}$

$= 76576.89$

$A =$ ₹ 76577

His salary will be after 5 years is ₹ 76577

6. Find the sum of the Geometric series $3+6+12+\cdots+1536$ PTA-3

Given geometric series

$3+6+12+\cdots+1536$

$a=3, r=\frac{t_2}{t_1}=\frac{6}{3}=2\,, l=1536$

$t_n=ar^{n-1}$ (n term is 1536)

$1536=3(2)^{n-1}$

$\frac{1536}{3}=2^{n-1}$

$512=2^{n-1}$

$2^9=2^{n-1}$

$9=n-1$

$n=10$

$S_n=a\left[\frac{r^n-1}{r-1}\right], r>1$

$S_{10}=3\left[\frac{2^{10}-1}{2-1}\right]=3(1024-1)=3(1023)$

$=\mathbf{3069}$

7. If $S_n=(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots n$ terms then prove that $(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}-\frac{y^2(y^n-1)}{y-1}\right]$ PTA-1

Given: $S_n=(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots+n$ terms 5M

Multiply by $(x-y)$

$(x-y)S_n=[(x-y)(x+y)+(x-y)(x^2+xy+y^2)+(x-y)(x^3+x^2y+xy^2+y^3)+\cdots+n$

$=[(x^2-y^2)+(x^3-y^3)+(x^4-y^4)+\cdots n$ terms$]$

$(x-y)S_n=[(x^2+x^3+x^4+\cdots+n$ terms $)-(y^2+y^3+y^4+\cdots n$ terms $)]$

$x^2+x^3+x^4+\cdots+n$ terms	$y^2+y^3+y^4+\cdots n$ terms
Here $a=x^2, r=\frac{x^3}{x^2}=x$	Here $a=y^2, r=\frac{y^3}{y}=y$

$S_n=\frac{a(r^n-1)}{r-1}$

$(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}\right]-\left[\frac{y^2(y^n-1)}{y-1}\right]$

$(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}-\frac{y^2(y^n-1)}{y-1}\right]$

Hence proved.

8. **Find the sum of the following series** PTA-5

(vi) $\mathbf{10^3 + 11^3 + 12^3 + \cdots + 20^3}$

$10^3 + 11^3 + 12^3 + \cdots + 20^3$

$= (1^3 + 2^3 + 3^3 + \cdots + 20^3) - (1^3 + 2^3 + \cdots + 9^3)$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$= \left[\frac{20(20+1)}{2}\right]^2 - \left[\frac{9(9+1)}{2}\right]^2$

$= \left[\frac{20(21)}{2}\right]^2 - \left[\frac{9(10)}{2}\right]^2$

$= [10(21)]^2 - [9(5)]^2$

$= (210)^2 - (45)^2$

$= 44100 - 2025$

$= \mathbf{42075}$

9. **Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, … ,24cm. How much area can be decorated with these colour papers?** PTA-1

Given: The size of 15 square colour papers are 10cm, 11cm, 12cm, … 24cm

The area of square $= a^2$

The colour paper decorated area

$= 10^2 + 11^2 + 12^2 + \cdots + 24^2$

$= (1^2 + 2^2 + \cdots + 24^2) \quad -(1^2 + 2^2 + 3^2 + \cdots + 9^2)$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$= \frac{24(24+1)(24\times2+1)}{6} - \frac{9(9+1)(2\times9+1)}{6}$

$= 4(25)(49) - 3(5)(19)$

$= 4900 - 285$

$= \mathbf{4615\ cm^2}$